PG – 384

First Semester M.Sc. Examination, January 2017
(CBCS)
MATHEMATICS
M 104T : Ordinary Differential Equations

Time : 3 Hours   Max. Marks : 70

Instructions : *1)* ***All*** *questions* ***have equal*** *marks.*
2) Answer ***any five*** *questions.*

1. a) Let $y_1, y_2, y_3 \ldots , y_n$ be a fundamental set of $L_n y = 0$. Then show that $z_1, z_2, z_3, \ldots , z_n$ also form a fundamental set of $L_n\, y = 0$ iff there exists a non singular matrix A such that

   $[z_1, z_2, z_3, \ldots , z_n]^T = A\,[y_1, y_2, y_3 \ldots , y_n]^T$.

   b) If the Wronskian of $y_1(x)$ and $y_2(x)$ is $3e^{4x}$ and if $y_1(x) = e^{2x}$, then find $y_2(x)$. **(9+5)**

2. a) State and prove Sturm's separation theorem.

   b) Let f(x) and g(x) be the two functions having n continuous derivatives in [a, b]. Then prove that

   $$\int_a^b g(x) L_n f(x)\, dx = \int_a^b f(x) L_n^* g(x)\, dx + \{[f,g](x)\}_a^b$$ **(7+7)**

3. a) If $y_1(x)$ is a solution of $y''(x) + a_1(x) y'(x) + a_2(x) y(x) = 0$. Then show that $y_2(x) = y_1(x)\, f(x)$ is also a solution of the same differential equation provided $f'(x)$ satisfies the equation $\left(y_1^2 y\right)' + a_1(x)\left(y_1^2\, y\right) = 0$.

   Also prove that $y_1(x)$ and $y_2(x)$ are linearly independent.

   b) Define a Lipschitz condition and test the validity of this condition with respect to y for $f(x, y) = \frac{\cos x}{x^2}(y + y^2)$; $|x - 1| < \frac{1}{2}$, $|y| \le 1$. **(7+7)**

4. a) Define the self-adjoint eigenvalue problem. Also prove that the eigenvalues of a self-adjoint eigenvalue problem are real.

   b) Show that the eigen functions corresponding to the distinct eigenvalues of a self-adjoint eigenvalue problem are orthogonal over the same interval. **(7+7)**

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5. a) Define ordinary, regular and irregular singular points of a differential equation and hence find the same for
$(1 - x^2)\ y'' - 2x y' + Ny = 0$, N is a constant.

b) Find the series solution of
$(x^2 - 1)\ y'' + 3x y' + xy = 0,\ y(0) = 4,\ y'(0) = 6.$ **(7+7)**

6. a) Obtain the general solution of the Laguerre differential equation.

b) Prove the following :

i) $xL_n'(x) = n\, L_n(x) - n\, L_{n-1}(x).$

ii) $L_n'(x) = -\sum_{m=0}^{n-1} Lm(x).$ **(7+7)**

7. a) Express an n^{th} – order differential equation as a system of first order differential equation and hence obtain it for $y''' - 14y'' + 10y' - 16y = 16t$.

b) Find the fundamental matrix and the general solution of $\underset{\sim}{X}'(t) = A\,\underset{\sim}{X}(t)$

where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix},\ \underset{\sim}{X}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}.$ **(7+7)**

8. a) Define the various types of critical points of the linear system

$$\frac{dx}{dt} = ax + by,$$

$$\frac{dy}{dt} = cx + dy,\ ad - bc \neq 0.$$

b) Locate the critical point and find the nature of the system

$$\frac{dx}{dt} = x + y,$$

$$\frac{dy}{dt} = 3x - y.$$

Also find the equation of phase path. **(7+7)**